

Mathematics. The science of patterns.

Math 220, Linear Algebra

Course Description

Designed for students planning studies in mathematics, engineering, computer science, and physics. Topics for the course include: systems of linear equations, vectors, matrices, linear transformations, vector spaces, eigenvalues/eigenvectors/eigenspaces/eigenbasis, orthogonalization and diagonalization.

1) Systems of linear equations

- a) Represent a linear system as an augmented matrix and use Gaussian elimination find the solution(s) to the system. Develop the notions of pivot columns, free variables, row echelon form, and reduce row echelon form of a matrix.
- b) For linear systems two and three unknowns develop an understand the geometry of a linear system and its solution(s).
- c) Use a computer algebra system (CAS) to solve linear system of any size using Gaussian elimination/row reduction.

2) Vectors

- a) Develop a visual understanding of vectors in \mathbb{R}^2 and \mathbb{R}^3 .
- b) Develop the notion of a linear combination of vectors algebraically and geometrically along with the span of a set of vectors.
- c) Develop the notions of the vector form of a line, the vector form of a plane, and the decomposition of the solution to a linear system as a linear combination of vectors.
- d) Develop thoroughly the notion of linear independence of a set of vectors. Emphasize the meaning of linear independence geometrically for vectors in \mathbb{R}^2 and \mathbb{R}^3 .

3) Matrices

- a) Develop the notion of a matrix times a vector as being a linear combination of the columns of the matrix using the vector entries as weights.
- b) Establish the connections between linear systems, vector equations, and matrix equations. Specifically the equivalent representation of a linear system as a vector equation or a matrix equation.
- c) Develop matrix multiplication by generalizing the matrix-vector product to the matrix-matrix product.
- d) Develop fully the rules of matrix algebra.
- e) Develop fully the notion of multiplicative inverses for matrices. When does matrix have an inverse and how is the inverse of a matrix determined?
- f) Develop competence in using a CAS to work with matrices, vectors, etc.
- g) Introduce the "Invertibility Theorem" establishing equivalences amongst matrices, vectors, and solutions to linear systems.

4) Linear Transformations

- a) Define a linear transformation from \mathbb{R}^m to \mathbb{R}^n . Establish that every linear transformation is represented by a matrix transformation. Develop the linear properties and algebraic operations on linear transformations. Define the kernel and range of a linear transformation.
- b) Investigate fully the geometric linear transformations in \mathbb{R}^2 and \mathbb{R}^3 : dilations/contractions, shears, rotations, reflections, rotations, and projections.
- c) Develop competence in using a CAS to work with linear transformations.

5) Vector Spaces

- a) Develop the notions of vector spaces, subspaces, basis, dimension for \mathbb{R}^n vector spaces. Emphasize geometry for \mathbb{R}^2 and \mathbb{R}^3 . Develop the notion of coordinates relative to a basis and how to change between different basis for a vector space.
- b) Develop the subspaces associated with a matrix – row space, column space, and null space. Include techniques for finding a basis for each of these spaces.

6) Determinants

Development of the basic properties of determinants and how to find the determinant of a matrix using a

CAS.

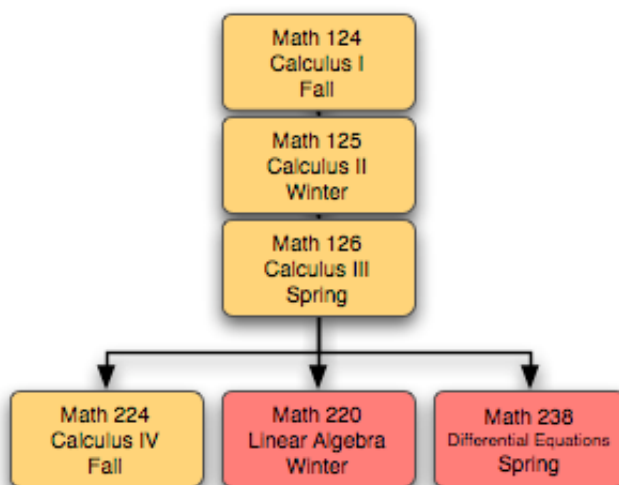
7) Eigenvalues, eigenvectors, eigenspaces, and eigenbasis

- Develop the notions of the eigenvalues, eigenvectors, and eigenspaces of a matrix and emphasize the geometry for 2 by 2 and 3 by 3 matrices.
- Define and develop the properties of the characteristic polynomial of a matrix.
- Develop the notion of diagonalization of a matrix and similarity.
- Develop competence in using a CAS to find eigenvalues and eigenvectors for a matrix, to determine the characteristic polynomial of a matrix, and to diagonalize a matrix.

8) Orthogonality

- Develop the notion of orthogonality for a set of vectors and an orthogonal/orthonormal basis for R^n .
- Develop the notion of orthogonal projections (into a subspace) and the Gram-Schmidt process for finding an orthogonal basis of a subspace.
- Develop the notion of orthogonal matrices, orthogonal complements, and orthogonal projections.
- Use orthogonality to develop least-squares solutions to inconsistent linear systems. Include the normal equations to the linear system. Use least-squares solutions to solve a data fitting problem.
- Develop competence in using a CAS to work with orthogonality concepts, Gram-Schmidt, and finding least-squares solutions to inconsistent linear systems.

Advanced math courses at WWCC:



Required Materials

- The textbook for the course is **Visual Linear Algebra**, 1st edition, Herman/Pepe/Schulz, John Wiley & Sons.
- Access to the computer algebra system *Mathematica*. WWCC's site license provides students their own personal copies of Mathematica 7. Install DVD's are available in the WWCC Bookstore for \$1.99.
- Engineering Computation paper for homework assignments. Available in the WWCC Bookstore for approximately \$2.50 for a 100-sheet pad.

Attendance

Attendance at every class session is expected. I understand absences are sometimes unavoidable and will work with students when such occasions arise. In the event of an absence occurring on the date of a scheduled exam or quiz, **prior** arrangements must be made in order to schedule another time to write the exam.

Cell Phones/PDAs



Our classroom is a **No Cell Phone/PDA environment**. Cell phones are to be silenced before class begins and put away. Cell phones/PDAs are not to be accessed for any reason during classtime. Text messaging is not allowed during class. Using a cell phone/PDA as a calculator is not acceptable - you should have a scientific calculator for use in the course. Develop the habit of silencing your phone when entering the classroom - I'm confident that everyone can manage to go 50 minutes without accessing their cell phone!

Our classroom is equipped with computers. We will use the computer resources at various times throughout the course for course activities. Other use of the computers is not to occur during class - no checking email, no instant message, no web browsing, no gaming, and no working on online homework assignments during class time.

Homework Assignments

Homework assignments comprised of textbook problems will given regularly and discussed in class.

Exams

There will be 4 exams: 3 exams scheduled during the quarter and a comprehensive final exam. Each exam will be composed of an in-class portion and a take-home portion that is to be completed outside of class and will require the use of Mathematica. The final exam is worth 200 points and the other 3 exams are worth 150 points each.

Grades

Grades for the course are computed by adding up the number of points earned and dividing by the total number of points possible in the course. Final grades are simply a function of the percentage of possible points earned. Let p be the percent of the possible course points earned by a student, the course grade is then given in the following table:

93% $\leq p \leq 100\%$ -> A
90% $\leq p < 93\%$ -> A-
87% $\leq p < 90\%$ -> B+
83% $\leq p < 87\%$ -> B
80% $\leq p < 83\%$ -> B-
77% $\leq p < 80\%$ -> C+
73% $\leq p < 77\%$ -> C
70% $\leq p < 73\%$ -> C-
67% $\leq p < 70\%$ -> D+
60% $\leq p < 67\%$ -> D
0% $\leq p < 60\%$ -> F
